



10<sup>th</sup> International Meeting of the  
Work Group on  
THE BEHAVIOUR OF HYDRAULIC  
MACHINERY UNDER STEADY  
OSCILLATORY CONDITIONS

Trondheim, Norway, June, 26.-28.2001

**“A NEW TOOL FOR THE SIMULATION OF DYNAMIC BEHAVIOUR OF  
HYDROELECTRIC POWER PLANTS”**

EPFL – Swiss Federal Institute of Technology of Lausanne  
IMHEF – Laboratory for Hydraulic Machines      LEME – Electrical Machines Laboratory  
C. Nicolet      A. Sapin  
J.-E. Prenat      J.-J. Simond  
F. Avellan

**ABSTRACT**

An analytical approach allowing modelling transient phenomena in pipes, valves, surge tank and Francis turbines based on impedance method is developed. These models are introduced in a software called “SIMSEN”, which simulates the behaviour of complex applications in electrical machines and power networks. This program is based on a modular structure, which enables the numerical simulation in transient modes of systems exhibiting arbitrary topologies. The numerical simulation for transient phenomena in hydropower plants with “SIMSEN” has the benefit of an algorithm that generates and solves automatically the main set of differential equations. This algorithm solves simultaneously the electrical, hydraulic and regulation equations assuring the interaction between the three parts of the systems. Three cases are investigated demonstrating the capability of these models to reproduce the “wave propagation”, the “mass oscillations” and the transient behaviour of Francis turbine.

**1/ INTRODUCTION**

The torque of a hydro electrical power plant constantly changes according to the network needs, generating variations of the turbine’s rotating speed. A regulation system acts on the guide vanes to keep the rotating speed constant. This manoeuvre engenders pressure fluctuations in the hydraulic system. This scenario shows interest to model the whole installation with the electric network, hydraulic plant and control systems to study the stability of the power plant.

A software called « SIMSEN » [1] has been developed for the simulation of power networks in transient or steady-state modes and speed drive systems including the regulation systems. This software includes the models of pipe, valve, surge tank and Francis turbine based on electric analogies. The behaviour of the whole power plant is described by a set of differential equations that are simultaneously integrated taking into account all the interactions between the electric, hydraulic and regulation systems.

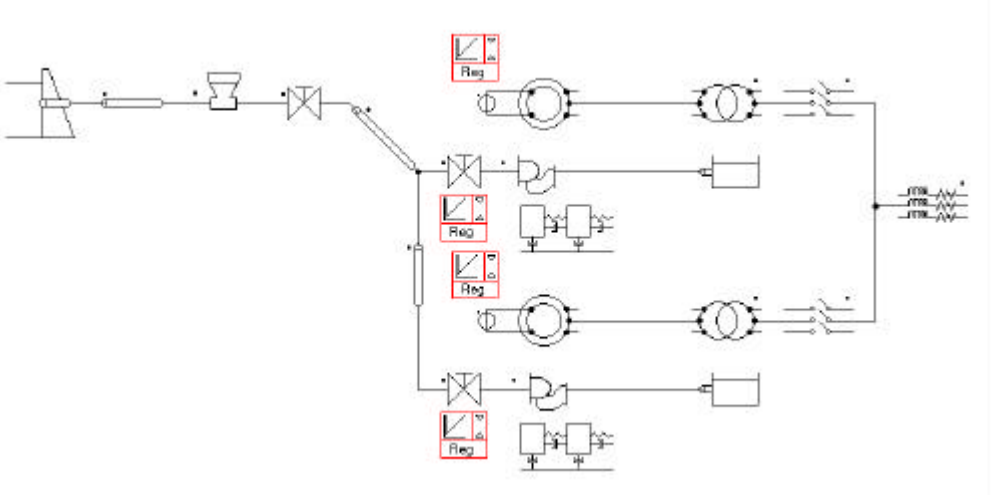
**2/ PRESENTATION OF “SIMSEN”**

This software is based on a modular structure, which enables the numerical simulation in transient or steady-state modes of systems with arbitrary topologies. It is composed of units,

each representing a specific cell in the network: electrical machine, mechanical system taking into account mechanical masses connected with damping and springs, transformer, voltage supply, transmission line, load, static converter, regulator. The originality of this package lies within its ability to simulate classical three phase power networks for the generation, transmission, distribution and networks involving semi-conductor units as well.

Each unit includes a set of differential equations based on the network element modelling. An original algorithm has been developed in order to generate the main set of differential equations solved by fourth order Runge-Kutta. The variable time-step used for the integration of the governing equations allows detecting the exact sequence of events like the on-off switching of semi-conductor or circuit-breakers phase on-off switching.

The hydraulic elements are modelled as an assembly of RLC components and the whole installation is modelled like an electrical network where the variables are: (1) the piezometric head  $H$  at the node and (2) the discharge  $Q$  through each components – respectively corresponding to voltage  $U$  and current  $i$ . Thus, the differential equations can be generated by “SIMSEN” using Kirchhoff’s law. The main differential set of equations contains electric, hydraulic and regulation equations that are solved simultaneously. An example of hydroelectric power plant, which will be created with “SIMSEN” is shown in figure 1.



**Figure 1:** Hydroelectric power plant modelled with SIMSEN including the hydraulic module.

**3/ MODELISATION**

The following hydraulic elements are modelled using electric analogies and are represented by RLC components:

- Tank
- pipe
- valve
- surge tank
- Francis turbine

**Pipe model**

The conservation of mass and motion equations applied to cross-flow section of fluid in a pipe give by linearisation:

$$\left. \begin{aligned} \frac{\partial H}{\partial t} + \underbrace{\frac{a^2}{gA}}_{1/C'} \frac{\partial Q}{\partial x} &= 0 \\ \frac{\partial H}{\partial x} + \underbrace{\frac{1}{gA}}_{L'} \frac{\partial Q}{\partial t} + \underbrace{\frac{I|Q|}{gDA^2}}_{R'} Q &= 0 \end{aligned} \right\} \quad (1)$$

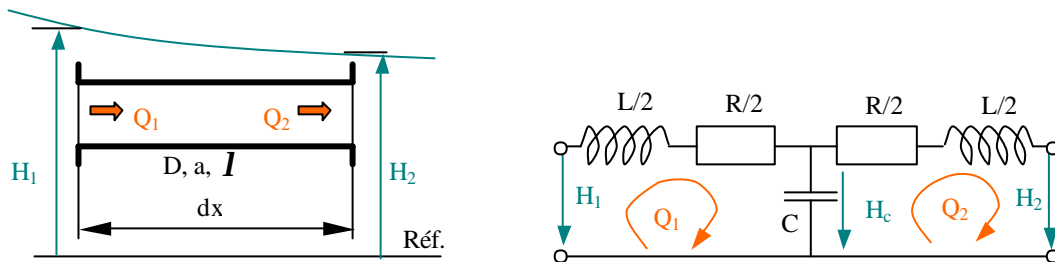
Equation (1) can be compared with the set of equations given in (2):

$$\begin{aligned} \frac{\partial U}{\partial t} + \frac{1}{C_e} \frac{\partial i}{\partial x} &= 0 \\ \frac{\partial U}{\partial x} + L_e \frac{\partial i}{\partial t} + R_e i &= 0 \end{aligned} \quad (2)$$

A capacitance  $C'$ , an inductance  $L'$  and a resistance  $R'$  per meter [2] can be identified as shown in equation (1) allowing to establish the equations (3):

$$\underbrace{dx C'}_C \frac{dH_c}{dt} = Q_1 - Q_2 \quad \Delta H_R = \underbrace{dx R'}_R Q_1 \quad \Delta H_L = \underbrace{dx L'}_L \frac{dQ_2}{dt} \quad (3)$$

**Figure 2** presents the electrical analogy for a pipe of length  $dx$ :



**Figure 2:** Slice of pipe modelled by electrical analogies.

The capacitive, inductive and resistive RLC terms respectively correspond to the storage, inertial and losses effects. A whole pipe is constituted of  $n$  circuits RLC.

### Diaphragm model

The losses trough a diaphragm are given by:

$$\Delta H_{nd} = \underbrace{\frac{K_d |Q_d|}{2}}_{R_d} Q_d \quad (4)$$

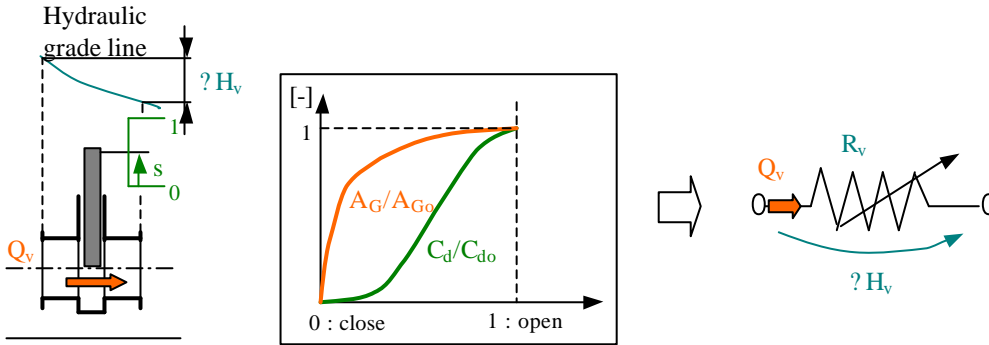
A diaphragm can represent all the singular losses that are described by a loss factor  $K_d$ .

### Valve model

The discharge equation of a valve where  $C_d$  is the discharge coefficient and  $A_G$  is the area of opening and  $s$  is the obturator course:

$$\Delta H_v = \underbrace{\frac{|Q_v|}{2g(C_d(s)A_G(s))^2}}_{R_v} Q_v \quad (5)$$

A valve can be corresponding to a variable resistance as shown in figure 3.



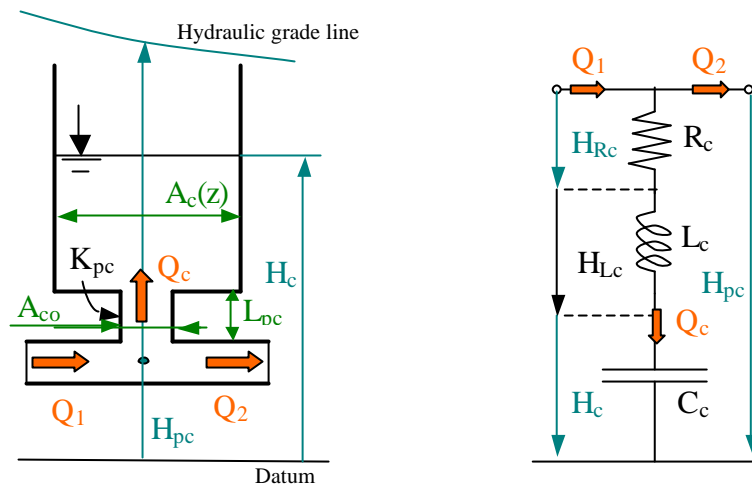
**Figure 3:** Valve modelled by electrical analogies.

### Surge tank model

A surge tank is described by singular losses at the inlet, a level variation and a inertia. According with hydraulic the following expression can be written:

$$(H_{pc} - H_c) = \underbrace{\frac{K_{pc}|Q_c|}{2gA_{co}^2}}_{R_c} Q_c \quad \underbrace{A_c \frac{dH_c}{dt}}_{C_c} = Q_c \quad L_c = \frac{L_{pc}}{gA_{co}} \quad (7)$$

Since these terms are related to the discharge  $Q_c$  incoming in the surge tank, then the three components  $R_c L_c C_c$  are placed in serie (figure 4).



**Figure 4:** Surge tank modelled by electrical analogies.

## Francis Turbine Model

There are two different approaches to model the Francis turbine (1) an analytical one and (2) an experimental one.

### Analytical model

The following analytical model was proposed by T.K. Nielsen [3], [4]:

$$T_m = \mathbf{r}Q\mathbf{h}_q \left[ R_1 C_1 \cos(\mathbf{a}_1) + R_1 C_1 \cot(\mathbf{b}_1) \frac{A_1}{A_1} - R_1^2 \mathbf{w} \right] \mathbf{h}_m \quad (8)$$

$$\underbrace{\frac{I_h}{\mathbf{r}g} \frac{dQ}{dt}}_{L_t} = H \underbrace{\left( 1 + \frac{s\mathbf{w}_R^2}{2gH_R} \right)}_{H_t} - \underbrace{\frac{s\mathbf{w}^2}{2g}}_{R_t} - \underbrace{\frac{H_R|Q|}{(Q_R\mathbf{k})^2}}_{R_t} Q \quad (9)$$

With the degrees of opening  $\kappa$  and the geometrical factor  $s$  :

$$\mathbf{k} = \frac{\sin(\mathbf{a}_1)}{\sin(\mathbf{a}_{1R})} \quad s = R_1^2 \left( 1 - \frac{R_1^2}{R_1^2} \right) \quad (10)$$

The second equation shows the presence of an inductance, a resistance and a source. The inductance  $L_t$  depends on the hydraulic inertia  $I_h$  and can be approximated by the following expression:

$$L_t = \int_l^{\bar{l}} \frac{1}{gA_t(l)} dl = \frac{I_h}{\mathbf{r}g} \quad (11)$$

### Experimental model

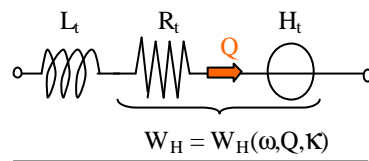
The experimental model based on mesured steady-state turbine characteristic is expressed as:

$$W_B(\mathbf{q}) = \frac{\mathbf{b}}{\mathbf{a}^2 + \mathbf{u}^2} \quad W_H(\mathbf{q}) = \frac{h}{\mathbf{a}^2 + \mathbf{u}^2} \quad \mathbf{q} = \tan^{-1}\left(\frac{\mathbf{u}}{\mathbf{a}}\right) \quad (12)$$

Where the dimensionless properties are given by:

$$\mathbf{a} = \frac{\mathbf{w}}{w_R} \quad \mathbf{u} = \frac{Q}{Q_R} \quad \mathbf{b} = \frac{T}{T_R} \quad h = \frac{H}{H_R} \quad (13)$$

Knowing the parameter  $Q$ ,  $\kappa$  and  $\omega$ ,  $T$  and  $H$  can be deduce from  $W_H$  and  $W_B$ . The characteristic  $W_H$  account the resistance and the source of the electrical model (figure 5).

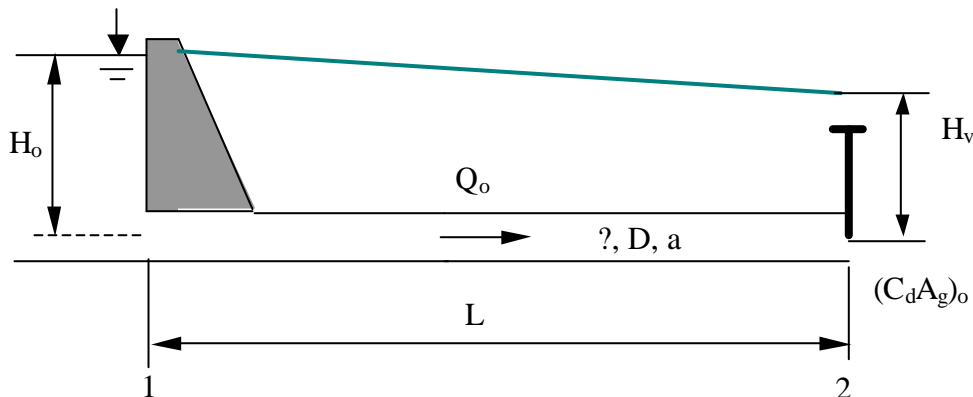


**Figure 5:** Simplified electrical model of the Francis turbine.

#### 4/ CASE STUDY

##### Elementary plant

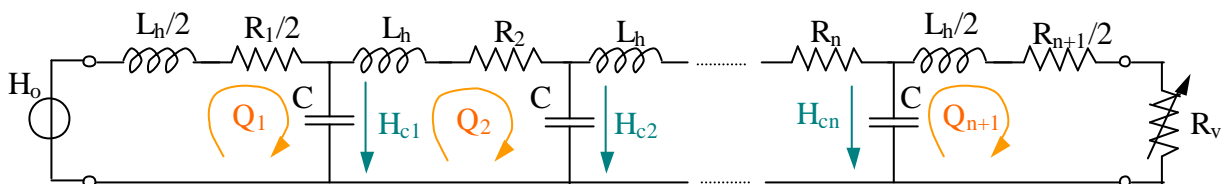
The first investigated case is taken from Wylie & Streeter [5]. The effect of the closure of a valve on a plant (figure 6) constituted by a tank, a pipe and the valve is studied.



**Figure 6 :** Configuration of the plant.

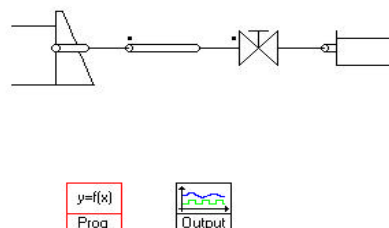
Tank	Pipe	Valve
$H_o = 150$ m	$L = 600$ m $D = 0.5$ m $a = 1200$ m/s $\gamma = 0.018$	$(C_d A_G)_o = 0.009$ m <sup>2</sup> Closure law: $t(t) = 1 - (t/2.1)^{0.75}$

**Table 1 :** Characteristic of the plant.

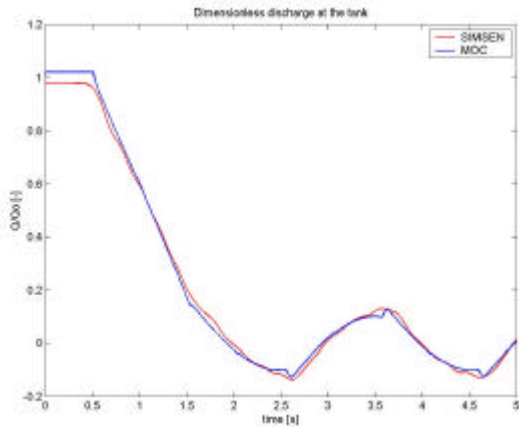


**Figure 7:** Electrical equivalent of the plant.

The transient behaviour of the plant is simulated with SIMSEN (figure 8). The comparison between simulation result obtained with (1) the method of characteristic and (2) SIMSEN is presented in figure 9. For the simulation the pipe is decomposed in 10 elements and the time step is 0.001 second.



**Figure 8:** Configuration created with "SIMSEN".



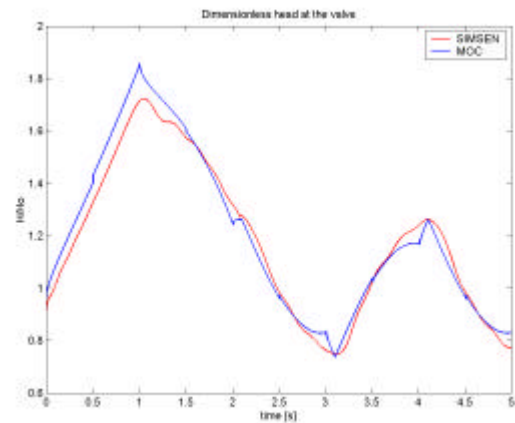
- a -

**Figure 9:** Comparison of the results of simulation with “SIMSEN” and method of characteristic (MOC).

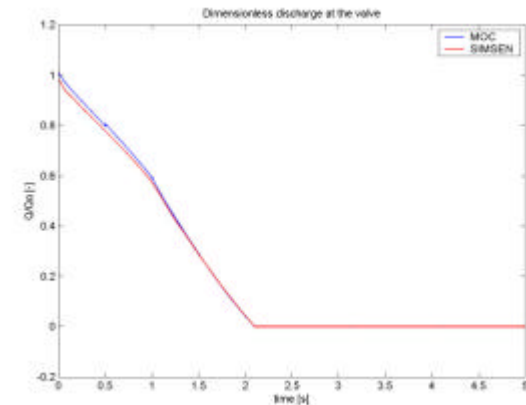
a. Evolution of the discharge  $Q$  in the tank.

b. Evolution of the head  $H_v$  at the valve.

c. Evolution of the discharge at the valve  $Q_v$ .

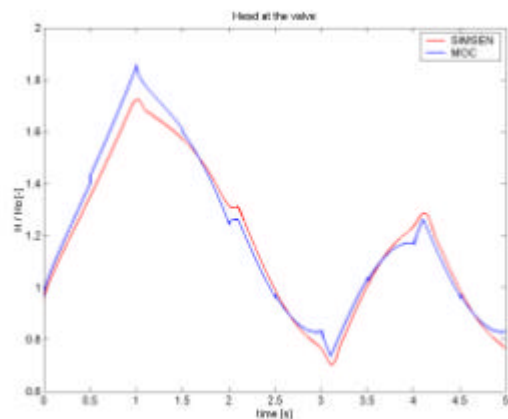


- b -

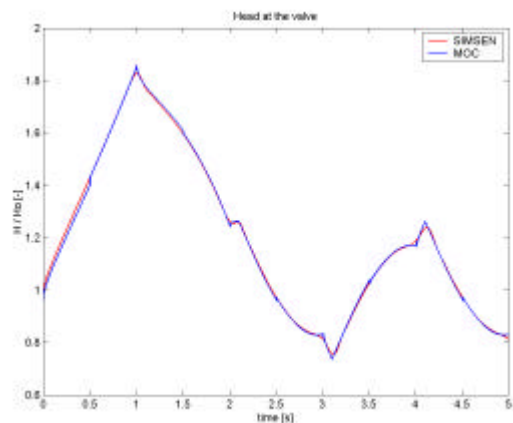


- c -

Other simulations have demonstrated, that, if the number of elements is increased to 30, the shape of the curve trends to match the one obtained by the method of characteristic (figure 10a). If the losses ( $\lambda = 0$ ) are neglected, the curve matches exactly the one obtained by method of characteristic (figure 10b). The electrical analogy correctly reproduce wave propagation phenomena, amplitudes of which are in good agreement.



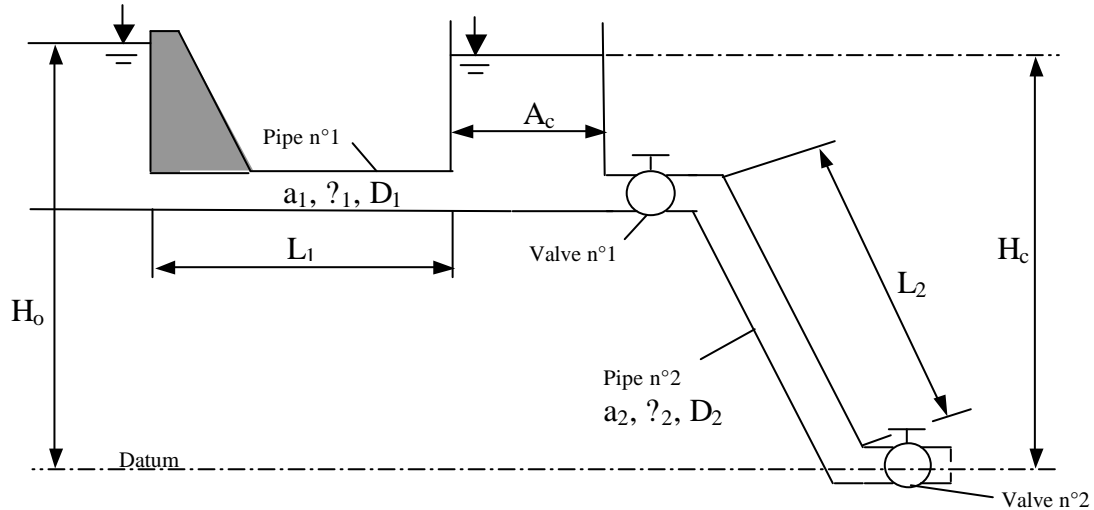
**Figure 10a:** Result of simulation obtained with 30 elements.



**Figure 10b:** Result of simulation with 30 element and neglected losses ( $\lambda = 0$ ).

### Plant with surge tank

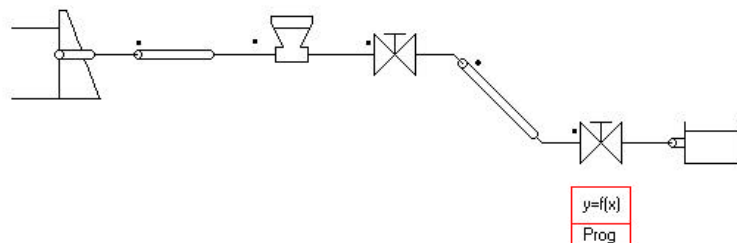
The second plant is constituted of a tank, a conducted gallery, a surge tank, 2 valves and a pipe (figure 11) whose the characteristics are provided in table 2. The effect of the closure of the second valve is investigated.



**Figure 11:** Plant with surge tank.

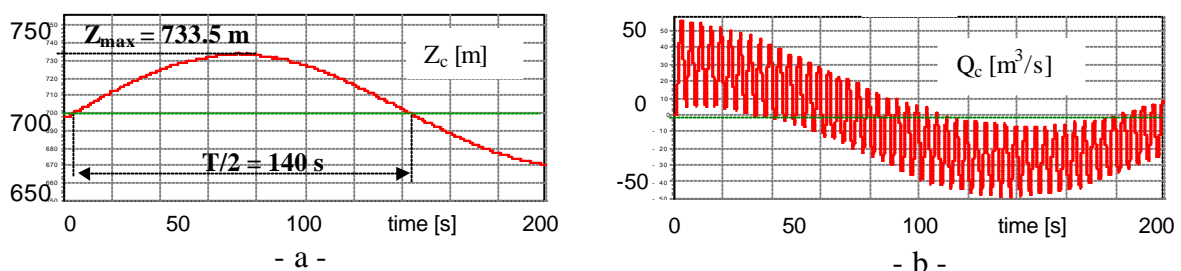
Tank	Pipe N°1	Surge tank	Valve N°1	Pipe N°2	Valve N°2
$H_0 = 700$ m	$L_1 = 5000$ m $D_1 = 3.57$ m $?_1 = 0.002$ $a_1 = 1100$ m/s	$A_c = 38.48$ m <sup>2</sup>	$C_d A_{G1} = 28.85$ m <sup>2</sup>	$L_2 = 1100$ m $D_2 = 2.52$ m $?_2 = 0.002$ $a_2 = 1100$ m/s	$C_d A_{G2} = 0.26$ m <sup>2</sup> Closing law : $t(t) = 1 - (t/2.1)^{0.75}$

**Table 2:** Characteristic of the plant with surge tank.



**Figure 12:** Configuration of the plant with the surge tank created with “SIMSEN”.

The results obtained for simulation of the dynamic behaviour of the system are presented in figure 13. A 10-element discretization is chosen for each pipe and the used time step is 0.001 second.



**Figure 13:** Result of the simulation of the dynamic behaviour of the plant with surge tank.  
a. The head  $Z_c$  in the surge tank. b. The discharge  $Q_c$  incoming in the surge tank.

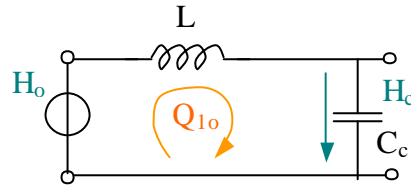


An analytic resolution of the governing equations - motion and continuity - applied to the junction between the pipe n°2 and the surge tank provides the oscillatory period and the amplitude of the mass oscillations (13).

$$\Delta z_{c \max} = \sqrt{\frac{L_1 A_1}{g A_c}} C_{10} = 34.5 \text{ m} \quad T = 2p \sqrt{\frac{L_1 A_c}{g A_1}} = 278 \text{ s} \quad (13)$$

The period and the amplitude of the head in the surge tank obtained by the simulation correctly matches with the analytical solution where friction factor is neglected. The visualisation of the discharge incoming in the surge tank present 2 frequencies. The lowest one corresponds to the mass oscillation and the highest one corresponds to a situation of a waterhammer in the pipe n°2, which is produced by the valve n°2 closure.

If the pipe n°1 is considered as an inductance and the surge tank as a capacity, they can be modelled as presented in figure 14.



**Figure 14:** Simplified model of the pipe N°1 and the surge tank.

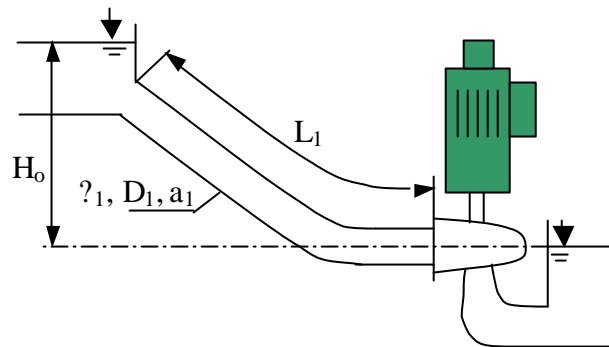
The value of the inductance of the pipe n°1 and the capacity of the surge tank introduced in the expression of the eigen period of the RL circuit (14), is identical to (13).

$$T_o = 2p \sqrt{LC} = 2p \sqrt{\frac{L_1 A_c}{g A_1}} \quad (14)$$

This case shows that these models can reproduce the wave propagation and the mass oscillations phenomena as well.

### Installation with Francis turbine

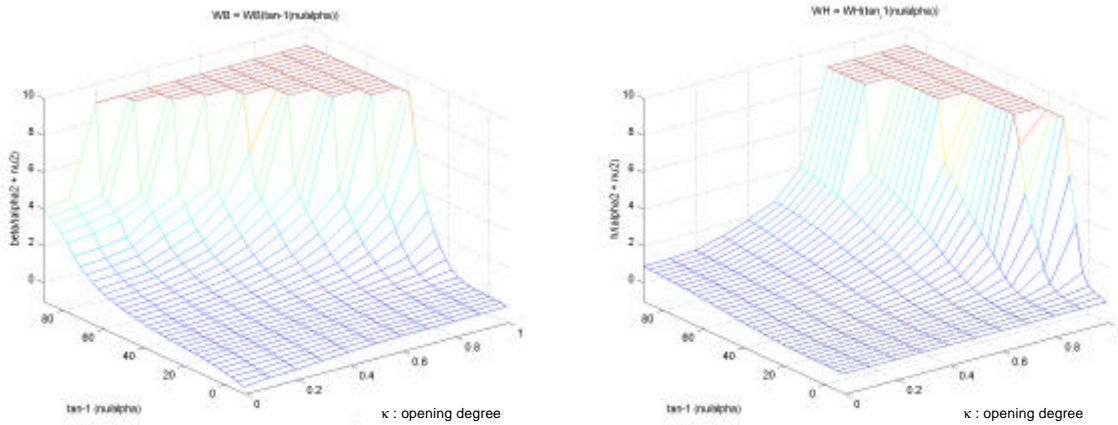
The last case is the model of a power plant constituted of a tank, a pipe, a Francis turbine and the generator. A 30% torque's decrease in a time period of 0.1 second is studied. The data of this example presented in table 3 and the characteristics of the turbine shown in figure 15 is taken from Wylie & Streeter [5].



**Figure 15:** Power plant with Francis turbine.

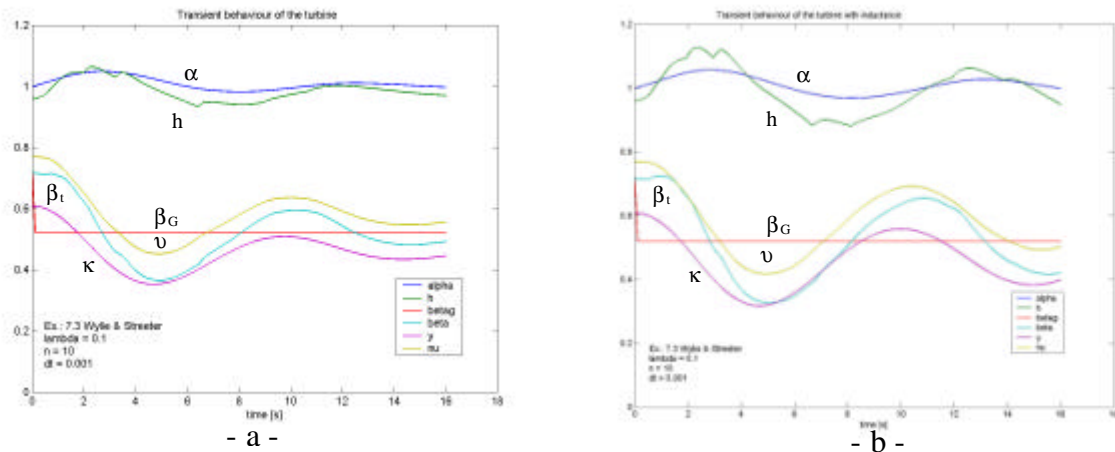
Tank	Pipe	Turbine	Generator	Regulator
$H_0 = 82$ m	$L = 125.27$ m $D = 5.48$ m $? = 0.1$ $a = 1250$ m/s	$H_{tR} = 82$ m $?_R = 20.94$ rd/s $Q_{tR} = 114$ m <sup>3</sup> /s $T_{tR} = 4.11e6$ Nm	$I_{t+g} = 1.5e6$ Kgm <sup>2</sup> $T_{gi} = 2.947e6$ Nm $T_{gf} = 2.14e6$ Nm $t_{ch} = 0.1$ s	$T_d = 3.7$ s $T_a = 0.325$ s $d = 0.18$

**Table 3 :** Characteristics of the power plant with Francis turbine.

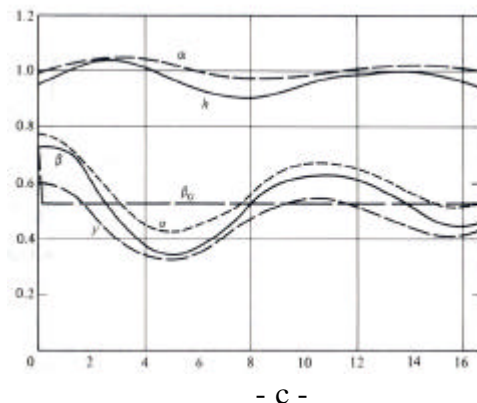


**Figure 16:** Characteristics  $W_B$  and  $W_H$  of the turbine.

From the dimensions and the equation (11), the inductance of the turbine is estimate to be  $L_t = 0.36$  s<sup>2</sup>/m<sup>2</sup>. The results of the simulation of the dynamic behaviour are presented in figure 17, (a) without the inductance and (b) with. The third graph shows the result obtained by [4]. The model of the turbine is not yet introduced in “SIMSEN” and the simulation is realised with MATLAB. The simulations obtained from MATLAB and the electric analogies match well as the result of [5]. The simulation featuring the inductance exhibits larger amplitudes.



**Figure 18:** Result of the simulation of the dynamic behaviour of Francis turbine.  
a. Without inductance  
b. With inductance  
c. Reference [4]



## 5/ CONCLUSION

Three case studies have demonstrated the possibility to reproduce the wave propagation and mass oscillation with electrical analogies implemented in “SIMSEN”. The models have been established for the pipe, the valve and the surge tank. The single element approach used does not need the Courant Friedrich Levy criteria to define the spatial step. This point with the modularity of SIMSEN is very important to simulate the dynamic behaviour of an hydraulic network with arbitrary topologies. Moreover, the parameters introduced in SIMSEN for hydraulic component are geometrical characteristic like length, diameter, wavespeed and loss factor. The electrical analogy is then transparent for the user.

The last example presented is a first step that shows the possibility to introduce a model of Francis turbine in “SIMSEN”. However a most accurate model for the turbine should be used in the future. For example a variable capacity can model the effect of the vapour torch in the diffuser. Then this software will be a useful tool to optimise the potential of large hydroelectric power plant.

## 6/ LIST OF SYMBOLS

a	wavespeed	[m/s]
A	area of pipe	[m <sup>2</sup> ]
A <sub>c</sub>	area of surge tank	[m <sup>2</sup> ]
A <sub>co</sub>	area of connecting pipe of surge tank	[m <sup>2</sup> ]
A <sub>G</sub>	area of opening of a valve	[m <sup>2</sup> ]
A <sub>t</sub>	area of turbine section	[m <sup>2</sup> ]
C	pipe capacitance <sup>1</sup>	[m <sup>2</sup> ]
C'	pipe capacitance per meter	[m]
C <sub>c</sub>	surge tank capacitance	[m <sup>2</sup> ]
C <sub>d</sub>	discharge coefficient	[-]
C <sub>e</sub>	electric capacitance	[F]
D	pipe diameter	[m]
g	terrestrial acceleration	[m/s <sup>2</sup> ]
h	dimensionless energy of turbine	[-]
H	piezometric head	[m]
H <sub>o</sub>	piezometric head of the tank	[m]
i	electric current	[A]
I <sub>h</sub>	hydraulic inertia of the turbine	[Kg/m <sup>4</sup> ]
I <sub>t+g</sub>	mechanical inertia of turbine and generator	[Kgm <sup>2</sup> ]
K <sub>d</sub>	singular loss coefficient of diaphragm	[-]
K <sub>pc</sub>	singular loss coefficient of connecting pipe of surge tank	[-]
L	pipe inductance <sup>1</sup>	[s <sup>2</sup> /m <sup>2</sup> ]
L'	pipe inductance per meter	[s <sup>2</sup> /m <sup>3</sup> ]
L <sub>e</sub>	electric inductance	[H]
L <sub>t</sub>	turbine inductance	[s <sup>2</sup> /m <sup>2</sup> ]
L <sub>pc</sub>	connecting pipe surge tank inductance	[s <sup>2</sup> /m <sup>2</sup> ]
Q	discharge	[m <sup>3</sup> /s]
R	pipe resistance <sup>1</sup>	[s/m <sup>2</sup> ]
R'	pipe resistance per meter	[s/m <sup>3</sup> ]
R <sub>d</sub>	diaphragm resistance	[s/m <sup>2</sup> ]

---

<sup>1</sup> In this paper the definition of capacitance, inductance, resistance do not correspond to literature's definition.

$R_t$	turbine resistance	[s/m <sup>2</sup> ]
$s$	obturator course	[-]
$T_t$	turbine torque	[Nm]
$T_g$	generator torque	[Nm]
$U$	voltage	[V]
$\alpha$	inlet runner angle	[rd]
$\beta$	outlet runner angle	[rd]
$\eta_m$	mechanical efficiency	[-]
$\eta_q$	volumetric efficiency	[-]
$\kappa$	opening degrees of guide vanes	[-]
$\lambda$	friction coefficient	[-]
$\theta$	abscissa of the turbine characteristic	[-]
$\rho$	fluid density	[-]
$\tau$	closing law	[-]
$\upsilon$	dimensionless discharge	[-]
$\omega$	rotating speed	[1/s]

## Indices

I:	inlet turbine
$\bar{I}$ :	outlet turbine
1:	inlet runner
$\bar{1}$ :	outlet runner
R:	rated

## 7/ REFERENCES

- [1] SAPIN, A. –1995 – Logiciel modulaire pour la simulation et l'étude des systèmes d'entraînement et des réseaux électriques. PhD thesis, EPFL, These n°1346.
- [2] FOX, J. A. – 1989 – Transients flow in pipes, open channels and sewers. Ellis Horwood Limited, Chichester.
- [3] NIELSEN, T. K. & RASMUSSEN, F.O. – 1992 – Analytic model for dynamic simulations of Francis turbines implemented in MOC. 16th Symposium of IAHR, Sao Paolo.
- [4] NIELSEN, T. K. – 1995 –Simulation of dynamic behaviour of governing turbines sharing the same electrical grid. 7th international meeting, Slovenia.
- [5] WYLIE, E. B. & STREETER, V.L. –1993 – Fluid transients in systems. Prentice Hall.