

Parametric study of Water column Separation in Francis Pump-turbine draft tube

Etude paramétrique de la rupture de colonne d'eau dans le diffuseur d'une pompe-turbine Francis

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The paper presents the modelling, simulation and analysis of the transient behaviour of a 340 MW pump-turbine in case of emergency shutdown in generation mode with particular attention to the possible draft tube water column separation. First, the model of a pumped storage power plant with simplified layout is setup with the EPFL software SIMSEN. This model includes a penstock feeding one 340MW pump-turbine with the related rotating inertia and a tailrace tunnel. The model of the tailrace tunnel allows for water column separation simulation. Thus, the related SIMSEN model is introduced and validations are briefly presented. Then, the influence of the tailrace tunnel length and diameter on the minimum pressure in the draft tube is investigated through a parametric study first without water column separation model. Finally, the simulation results of the transient behaviour of the pump-turbine with downstream water column separation are presented for different degree of severity triggered by the submergence and the tailrace tunnel dimensions taken as parameters.

Ce papier présente la modélisation, la simulation et l'analyse des régimes transitoires induit par une pompeturbine de 340 MW dans le cas d'un arrêt d'urgence en mode turbine avec une attention particulière portée au risque de rupture de colonne liquide pouvant survenir dans le diffuseur de la turbine. Tout d'abord, l'aménagement présentant une configuration simplifiée est modélisé au moyen du logiciel SIMSEN développé par l'EPFL. Ce modèle prend en compte la conduite forcée, une pompe-turbine de 340MW avec les inerties en rotation, et la galerie de fuite, dont le modèle permet de modéliser la rupture de colonne liquide. Le modèle de rupture de colonne liquide implémenté en SIMSEN et sa validation sont brièvement présentés. Ensuite, l'influence de la longueur et du diamètre de la galerie de fuite sur les valeurs minimums de pression obtenues en cas d'arrêt d'urgence en mode turbine est étudiée au travers d'une analyse paramétrique sans le modèle de rupture de colonne liquide. Finalement, les résultats de simulation d'arrêt d'urgence en mode turbine induisant une rupture de colonne liquide dont l'apparition est induite par la réduction de l'enfoncement et la modification des dimensions de la galerie de fuite sont présentés.

Key words Transient analysis, pump-turbine, cavitation, water hammer.

I INTRODUCTION

Pumped storage power plants are subjected to transient operation resulting from units start-up, normal shutdown, emergency shutdown, power failure etc. Transient analysis are carried out at early stage of a project to define the hydraulic layout of the power plant and check the compatibility of the hydraulic machines transients with the foreseen adduction system. Special care has to be paid for high head projects, as they usually leads to long penstock, high rotational speed, with low inertia and short mechanical time constant and low specific speed pump-turbines [1], [2]. The case of pump-turbine emergency shutdown in generation mode is usually one of the most critical cases with respect to the maximum and minimum pressure induced in the piping system. Indeed, the pump-turbine reaches transient runaway and faces unstable behaviour related to the so-called S-shape of the pump-turbine characteristic [3], [4], [5], [6]. During runaway and guide vane closure, the transient operating point of the pump-turbine goes from the normal turbine operation in the first quadrant, to turbine brake and then to reverse pumping in the fourth quadrant. The excursion from normal turbine operation to reverse pumping being achieved in very short time, it leads to large and fast variation of discharge and thus generate high positive pressure wave in the penstock and negative pressure wave in the draft tube. If the penstock can be designed to withstand extreme value of pressure, the minimum pressure in the draft tube has to be addressed with particular attention to ensure sufficient safety margin to prevent from water column separation.

The paper presents the modelling, simulation and analysis of the transient behaviour of a 340 MW pump-turbine in case of emergency shutdown in generation mode with particular attention to the possible draft tube water column separation. First, the model of a pumped storage power plant with simplified layout is setup with the EPFL software SIMSEN. This model includes a penstock feeding one 340MW pump-turbine with the related rotating inertia and a tailrace tunnel. The model of the tailrace tunnel allows for water column separation simulation. Thus, the related SIMSEN model is introduced and validations are briefly presented. Then, the influence of the tailrace tunnel length and diameter on the minimum pressure in the draft tube is investigated through a parametric study first without water column separation model. Finally, the simulation results of the transient behaviour of the pump-turbine with downstream water column separation are presented for different degree of severity triggered by the submergence and the tailrace tunnel dimensions taken as parameters.

II HYDROACOUSTIC MODELLING

II.1 Modelling of Hydraulic Components

By assuming uniform pressure and velocity distributions in the cross section and neglecting the convective terms, the one-dimensional momentum and continuity balances for an elementary pipe filled with water of length dx , cross section A and wave speed a , see Fig. 1, yields to the following set of hyperbolic partial differential equations [7]:

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{a^2}{gA} \cdot \frac{\partial Q}{\partial x} = 0 \\ \frac{\partial h}{\partial x} + \frac{1}{gA} \cdot \frac{\partial Q}{\partial t} + \frac{\lambda |Q|}{2gDA^2} \cdot Q = 0 \end{cases} \quad (1)$$

The system (1), where Q is the discharge and h is the piezometric head, is solved using the Finite Difference Method with a 1st order centered scheme discretization in space and a scheme of Lax for the discharge variable. This approach leads to a system of ordinary differential equations that can be represented as a T-shaped equivalent scheme [8], [9], [10] as presented in Fig. 2. The RLC parameters of this equivalent scheme are given by:

$$R = \frac{\lambda \cdot |Q| \cdot dx}{2 \cdot g \cdot D \cdot A^2} \quad L = \frac{dx}{g \cdot A} \quad C = \frac{g \cdot A \cdot dx}{a^2} \quad (2)$$

where λ is the local loss coefficient. The hydraulic resistance R , the hydraulic inductance L , and the hydraulic capacitance C correspond respectively to energy losses, inertia and storage effects. The model of a pipe of length L is made of a series of n_b elements based on the equivalent scheme of Fig. 2. The system of equations relative to this model is set-up using Kirchoff laws. The model of the pipe, as well as the model of

valve, surge tank, Francis turbine, etc, are implemented in the EPFL software SIMSEN, developed for the simulation of the dynamic behavior of hydroelectric power plants, [9], [11]. The time domain integration of the full system is achieved in SIMSEN by a Runge-Kutta 4th order procedure.

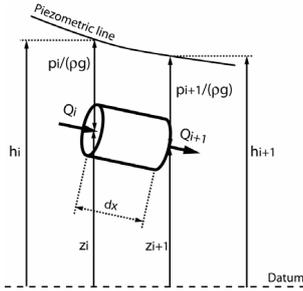


Fig. 1 Elementary hydraulic pipe of length dx.

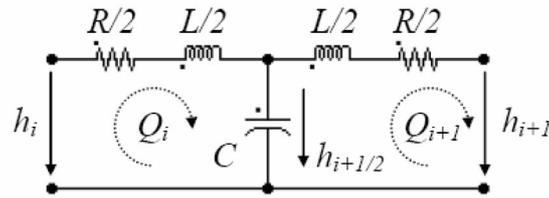


Fig. 2 Equivalent circuit of an elementary pipe of length dx.

As presented in Table 1, the modeling approach based on equivalent schemes of hydraulic components is extended to all the standard hydraulic components such as valve, surge tanks, air vessels, cavitation development, Francis pump-turbines, Pelton turbines, Kaplan turbines, pump, etc, see [9].

Component	Hydraulic scheme	Electrical equivalent scheme	Parameters
Generalized pipe			$R = \frac{dx\lambda Q }{2gDA^2} \quad R_v = \frac{\mu}{\rho g A dx}$ $L = \frac{dx}{gA} \quad C = \frac{dxgA}{a^2}$
Valve			$R_v = \frac{K_v(\alpha) Q }{2gA^2}$
Surge tank			$R_d = \frac{K_d(Q) Q }{2gA^2}$ $C_{ST} = A_{ST}(h_c)$
Francis pump-turbine			$H = H(W_H(y, Q, N))$ $T = T(W_T(y, Q, N))$ $R_t = R_t(W_T(y, Q, N))$ $L_t = \frac{l_{equ}}{gA}$
V_g : volume of gas [m ³] W_H : turbine head characteristic [-] l_{equ} : turbine equivalent length [m] h_g : pressure of gas [m] W_T : turbine torque characteristic [-] μ : viscosity of the fluid or material [Pa·s]			

Table 1 Modeling of hydraulic components with related equivalent schemes.

II.2 Water column separation modeling

The free gas content of water significantly reduces the wave speed in pressurized pipelines, see [12], [13] and [14]. Wylie [13] derived wave speed in homogenous liquid free gas mixture characterized by an initial void fraction α_0 defined for a reference absolute pressure p_0 and leads to the following equation:

$$a = \frac{a_o}{\sqrt{1 + \frac{p_o \alpha_o a_o^2}{\rho g^2 (h - Z - Hv)^2}}} \quad (3)$$

where :

a_o	[m/s]	Wave speed in liquid
p_o	[Pa]	Reference absolute pressure
α_o	[-]	Initial void fraction
ρ	[kg/m ³]	Liquid density
g	[m/s ²]	Gravitational acceleration
h	[m]	Piezometric head
Z	[m]	Pipe elevation
Hv	[m]	Vapour pressure head

Thus, the wave speed in liquid gas mixture is function of the local piezometric head. Figure 3 shows the wave speed evolution as function of the absolute gas partial pressure ($h-Z-Hv$) and of the initial void fraction α_o . The non-linear equation (3) is introduced in the equation set (1) for time domain simulation so that the wave speed is local piezometric head dependant $a=a(h_i)$, similar to Himr and Haban [15]. During water column separation, the local piezometric head drops to very low values and if the local pressure becomes negative due to numerical inaccuracy, the equation (3) leads to an increase of the wave speed, see [16]. Therefore, the wave speed is bounded to a minimum value defined as “ a_{min} ” to avoid numerical instability. The minimum wave speed “ a_{min} ” being defined *a priori*.

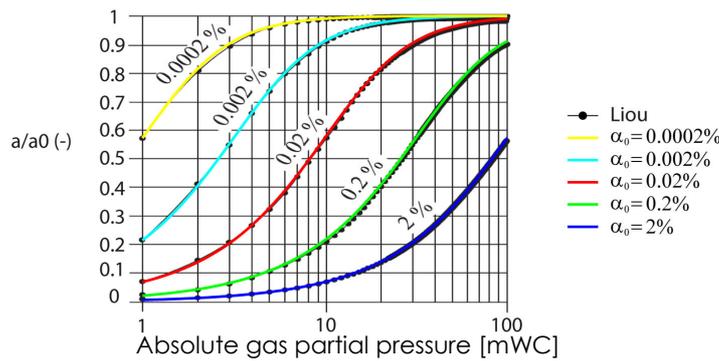


Fig. 3 Wave speed ratio as function of the initial void fraction α_o and of the absolute gas partial pressure ($h-Z-Hv$) (adapted from Liou [13]).

During water column separation, the bubbly liquid vapor mixture is subjected to dissipation resulting from phase changes. This dissipation is modeled by a thermodynamic damping μ'' also known as the bulk viscosity or fluid second viscosity, see Pezzinga [17]. This thermodynamic damping is introduced in the numerical scheme by means of an additional thermodynamic resistance R_{th} in series with the capacitance, see Alligné *et al.* [18], and defined as follows:

$$R_{th} = \frac{\mu''}{A \rho g dx} \quad (4)$$

The capacitance modelling the compressibility and wall deformation effects in series with thermodynamic resistance corresponds to a Kelvin-Voigt rheological model [16]. The modified equivalent scheme of an elementary pipe with water column separation is presented in Fig. 4 where the capacitance is pressure dependant. This water column separation model is implemented in SIMSEN and was validated with experimental data and also compared with Method of Characteristic, MOC, with Discret Gas Cavity Model, DGCM, results, see [16]. The test case, see [19], is a 37.23 meters long pipe of diameter 0.0221 meters with fast downstream valve closure inducing water column separation as presented in Fig. 5. Fig. 6 presents the comparison between SIMSEN simulation results, MOC-DGCM simulation results and experimental results where it can be noticed that both simulation results shows good agreements with experimental data.

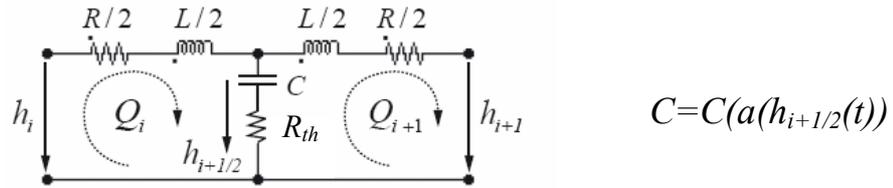


Fig. 4 Equivalent scheme of an elementary pipe with water column separation including pressure dependency of the wave speed and thermodynamic damping.

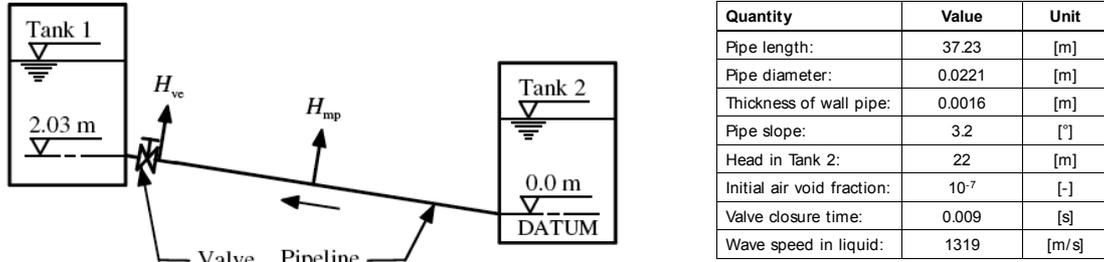


Fig. 5 Test case experimental apparatus, from [19].

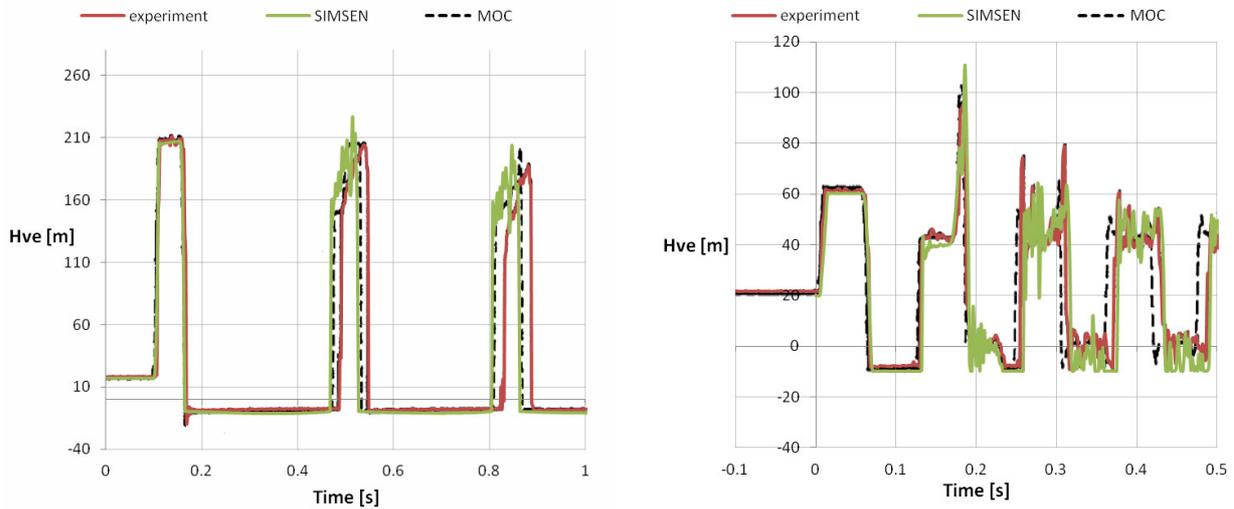


Fig. 6 Comparison simulation results obtained with SIMSEN free-gas mixture model and MOC-DGCM [20] model with experimental results at the downstream valve for $C_0=1.4$ m/s (left) and for $C_0=0.3$ m/s (right), see [16]

III ANALYTICAL EXPRESSION OF WATER HAMMER

The water hammer amplitude in case of fast flow velocity variation ΔC in pipe ($T_c < 2L/a$) can be derived from momentum and continuity equations and lead to the following equation [12]:

$$\Delta H = -\frac{a \cdot \Delta C}{g} \quad (5)$$

In case of full valve closure, the flow velocity variation upstream the valve is $\Delta C = -C_o$ and leads to the so-called Joukowski equation:

$$\Delta H_{\max} = \frac{a \cdot C_o}{g} \quad (6)$$

while the water hammer amplitude downstream the valve is given by:

$$\Delta H_{\max} = -\frac{a \cdot C_o}{g} \quad (7)$$

Figure 7 illustrates the positive and negative pressure waves propagating in a system with fast valve closure with upstream and downstream pipes of same size.

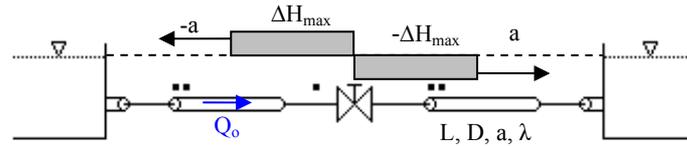


Fig. 7 Water hammer induced by fast ($T_c < 2L/a$) valve closure between two pipes of same size.

If the flow variation is linear, and the valve closure time T_c is longer than the reflection time of the pipe ($T_c > 2L/a$), the water hammer amplitude can be estimated with the Michaud equation [12]:

$$\Delta H = \Delta H_{\max} \cdot \frac{2L}{a} \cdot \frac{1}{T_c} = \frac{Q_0}{A} \cdot \frac{2L}{T_c} \quad (8)$$

From the Michaud equation, it can be derived that the water hammer amplitude:

- increases with initial discharge in the pipe Q_0 ;
- increases with pipe length L ;
- reduces with increased pipe cross section A ;
- reduces with valve longer closer time T_c .

IV CASE STUDY

The case study is a hydroelectric power plant with a simplified layout as illustrated in Fig. 8 made of an upstream reservoir with constant water level, a penstock of about 1100meters, a pump-turbine of 340MW which nominal parameters are given in Table 2, and a tailrace tunnel of 150 meters long and 4.7 meters of diameter. The pump-turbine is modelled by the 4 quadrant characteristics given by the guide vane opening y , the speed factor $N11$, the discharge factor $Q11$ and the torque factor $T11$, and the inertia of the total rotating masses J .

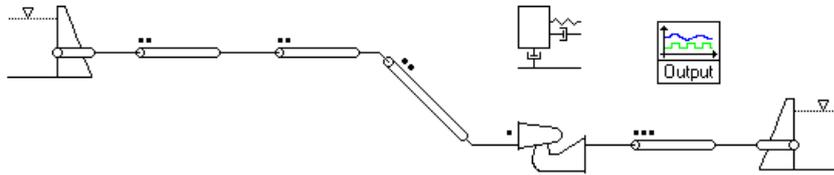


Fig. 8 SIMSEN model of the pump-turbine case study with simplified layout.

H_R [m]	Q_R [m ³ /s]	N_R [rpm]	P_R [MW]	ν [-]	J [kgm ²]
440	86	428.6	340	0.26	$1.5 \cdot 10^6$

Table 2 Rated values of the pump-turbine of Fig. 8.

V FRANCIS PUMP-TURBINE TRANSIENT AND POSSIBLE DRAFT TUBE WATER COLUMN SEPARATION

V.1 Pump-turbine transient in case of emergency shutdown in generating mode

Fig. 9 presents the simulation results obtained with SIMSEN for the transient behaviour of the pump-turbine of the system shown in Fig. 8 in case of emergency shutdown in generating mode occurring at $t=1s$ and with guide vane closure in $T_c=25s$. Fig. 10 shows the transient operating point experienced by the pump-turbine during the emergency shutdown in the $[N11-Q11]$ plain with the guide vane opening as parameter. One can notice that after disconnection from the grid, the pump turbine experiences rotational speed rise inducing an increase of $N11$ and thus a fast discharge reduction leading to a negative discharge due to the so-called S-shape of the pump-turbine characteristic [1], [2]. The fast discharge reduction produces positive water hammer pressure wave in the penstock and negative pressure wave in the tailrace tunnel resulting in a

net head increase. During the guide vane closure, the pump-turbine experiences two times unstable behaviour with excursions in the fourth quadrant corresponding to reverse pumping with negative discharge and positive rotational speed.

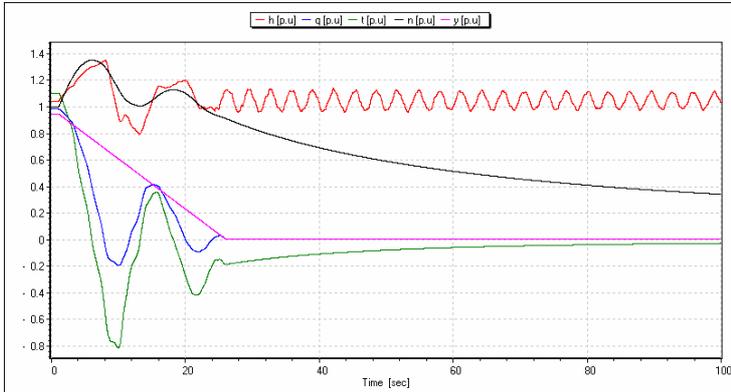


Fig. 9 Pump-turbine transient behaviour in case of emergency shutdown in generating mode with h the net head, q the discharge, t the torque, n the rotational speed and y the guide vane opening all related to rated values.

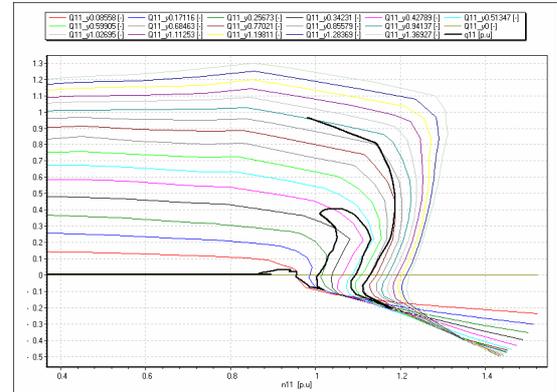


Fig. 10 Pump-turbine transient operating point in case of emergency shutdown in generating mode in the N11-Q11 plain.

V.2 Comparison with a simplified model with a valve

Fig. 11 shows the case study of Fig. 8 where the pump-turbine is replaced by a valve. The valve is closed with the same closing time as the pump-turbine guide vane closure time, and initial conditions of discharge are set identical to the case with pump-turbine. Fig. 12 presents the comparison of simulation results obtained for the discharge and the net head of the pump-turbine of Fig. 8 and of the valve of Fig. 11. It can be noticed that for both systems, the overall discharge variation is identical and corresponds to the initial discharge. However, the pump-turbine reverse flow induced by the runaway leads to much larger maximum transient value of the net head of 1.36p.u while the valve closure leads only to 1.1 pu. For the case with the pump-turbine, the discharge maximum variation is about 1.2 p.u in 9s, while with the valve, the discharge variation corresponds to 1 p.u in 25s, which explains the difference of net head variation between the valve and the pump-turbine. Fig. 13 shows the comparison of the time evolution of the pressure at the draft tube inlet obtained with the pump-turbine and with the valve. For the case with pump-turbine, the pressure in the draft tube drops to atmospheric pressure ($H_{p1}=0\text{mWC}$), while with the valve the pressure drops only to 17mWC above the atmospheric pressure. This demonstrates that pump-turbine induced water hammer cannot be approximated with a simplified model with the valve which is by nature not capable of reproducing reverse flow effects of the pump-turbine transient.

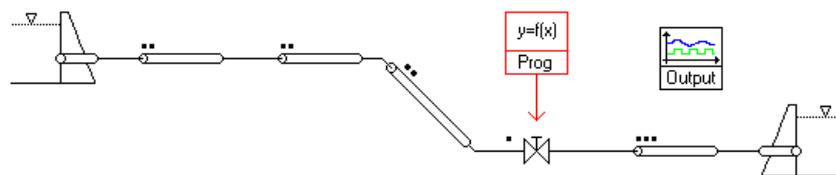


Fig. 11 SIMSEN model of same case study as Fig. 8 with a valve instead of the pump-turbine.

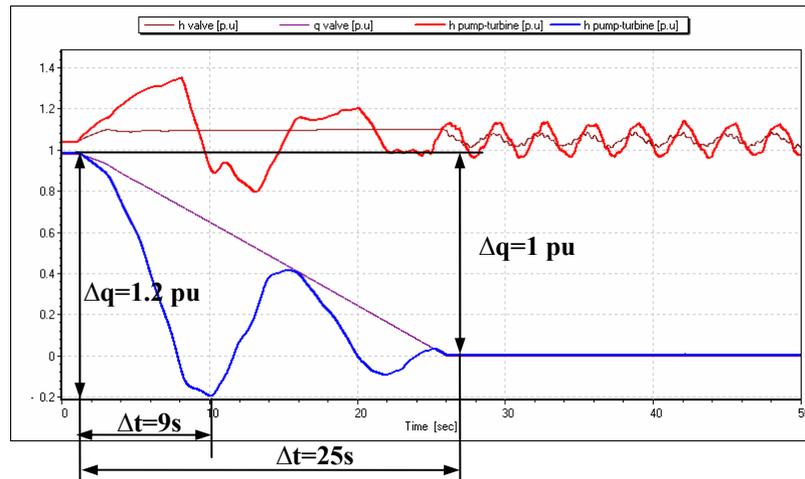


Fig. 12 Comparison of the time evolution of the pump-turbine discharge and net head of the system of Fig. 8 with the discharge and net head of the valve of system of Fig. 11 all related to rated pump-turbine values.

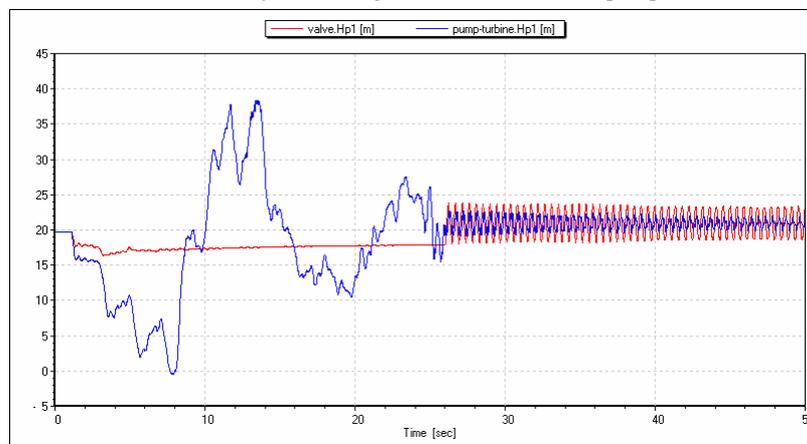


Fig. 13 Comparison of the time evolution of the pressure at the inlet of the draft tube obtained with the pump-turbine and with the valve.

V.3 Influence of the tailrace water tunnel dimensions

The influence of tailrace tunnel dimensions is evaluated by comparing the simulation results obtained with the system with pump-turbine of Fig. 8 without the model of water column separation in case (i) that the cross section of the tailrace tunnel is reduced to 0.72 times the original cross section and in case (ii) that the tailrace tunnel length is doubled. It can be noticed that:

- if the tailrace cross section is reduced to 0.72% of the original value, the pressure drop ΔH in the draft tube increases from 20mWC to 26mWC which ratio is 77% is similar to cross section ratio $\Delta H / \Delta H_{ref} \approx A_{ref} / A$ which is in good agreement with Eq. (7);
- if the tailrace tunnel length is doubled, the pressure drop ΔH increases from 20mWC to 38mWC, which ratio is similar to $\Delta H / \Delta H_{ref} \approx L / L_{ref}$ also in good agreement with Eq. (7).

This means that long tailrace tunnels results in large pressure drops in pump-turbine draft tube and increases the risk of water column separation, and that increasing the tailrace tunnel diameter, can help to mitigate possible low pressure problems in the draft tube.

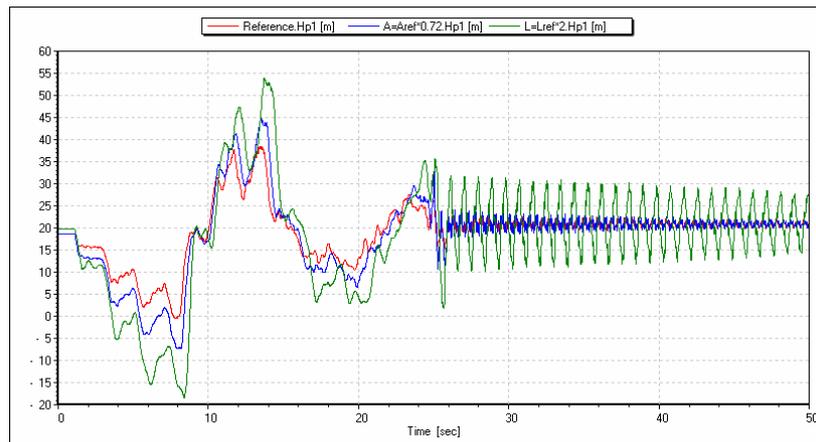


Fig. 14 Time evolution of the pressure at the inlet of the draft tube obtained with the pump-turbine in case of emergency shutdown in generating mode in case (i) the tailrace tunnel cross section is reduced to 72% of the original cross section and in case (ii) the length of the tailrace tunnel is doubled.

V.4 Draft tube water column separation

The transient behaviour of the system with the pump-turbine of Fig. 8 is simulated with the model of water column separation described in the chapter II.2 for the tailrace tunnel in case of emergency shutdown, ESD, in generating mode. The water column separation is induced by modifying the submergence and the tailrace tunnel length as follows:

- Case A) the tailrace water level is reduced by 15m to obtain a minimum negative pressure in the draft tube 5mWC below the vapour pressure during ESD, when using the classical water hammer model without column separation;
- Case B) the tailrace water level is reduced by 20m to obtain a minimum negative pressure in the draft tube 10mWC below the vapour pressure during ESD, ditto;
- Case C) the tailrace water level is reduced by 20m and the tailrace water tunnel length is doubled to obtain a minimum negative pressure 28mWC below the vapour pressure during ESD, ditto.

The simulation results obtained for the cases A) to C) with and without the water column separation model are respectively presented in Fig. 15 to Fig. 17. It can be noticed that, as expected, for the three cases, water column separation occurs and then leads to vapour cavity collapse that results in sudden pressure rise which maximum amplitudes corresponds to 65mWC, 208mWC to 366mWC above the atmospheric pressure respectively for the cases A), B) and C). For the case C), the maximum pressure obtained in the draft tube would correspond to 82% of the nominal head, and thus would considerably jeopardize the power plant integrity.

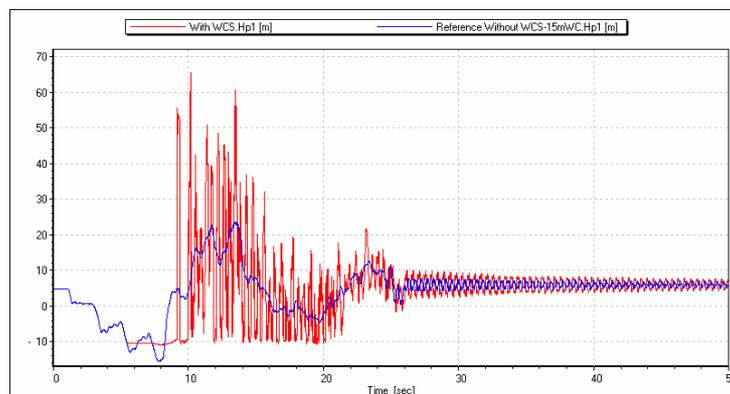


Fig. 15 Time evolution of the pressure at the inlet of the draft tube obtained with the pump-turbine in case of emergency shutdown in generating mode with and without water column separation model (WCS) if the tailrace water level is reduced by 15m (case A).

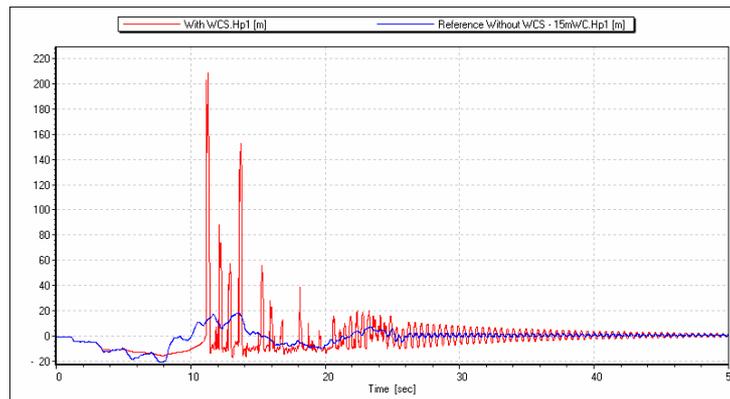


Fig. 16 Time evolution of the pressure at the inlet of the draft tube obtained with the pump-turbine in case of emergency shutdown in generating mode with and without water column separation model (WCS) if the tailrace water level is reduced by 20m (case B).

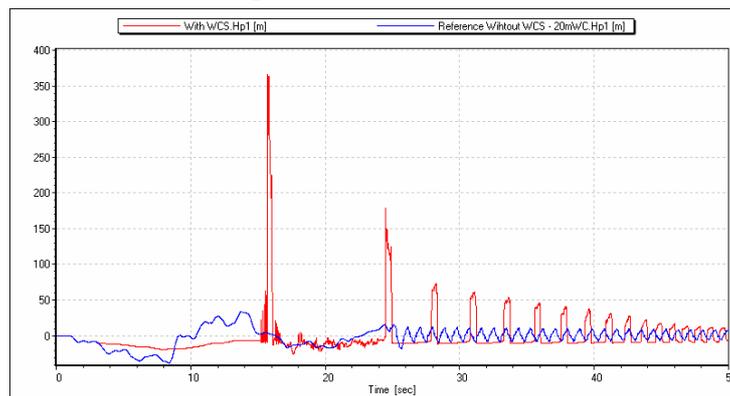


Fig. 17 Time evolution of the pressure at the inlet of the draft tube obtained with the pump-turbine in case of emergency shutdown in generating mode with and without water column separation model (WCS) if the tailrace water level is reduced by 20m and the length of the tailrace tunnel is doubled (case C).

VI CONCLUSIONS

This paper presents the modelling, simulation and analysis of possible water column separation in pump-turbine draft tube which could occur during emergency shutdown in generating mode using the simulation software SIMSEN. The model of water column separation was validated with a test case of water hammer in pipe induced by fast downstream valve closure. Then, a pump-turbine test case with simplified layout is presented. The simulation results obtained in case of emergency shutdown in generating mode shows the influence of the S-shape of the 4 quadrant characteristic on the time evolution of the discharge which becomes negative due to excursions in the reverse pumping fourth quadrant. The comparison with simulation results obtained with oversimplified model where the pump-turbine is replaced by a valve demonstrates that this approach is not suitable as valve characteristic is not capable to reproduce a rapid discharge drop resulting from the turbine runaway. Moreover, the S-shape of the characteristics strongly depends on the specific speed of the machine and also on the manufacturer's design. Therefore, realistic 4 quadrant characteristics and sufficient safety margin with respect to the draft tube minimum pressure should be considered during feasibility study to prevent water column separation. The parametric study carried out on the tailrace tunnel dimensions shows that a pressure drop in the draft tube is proportional to the tailrace length, and inversely proportional to the tailrace cross section. These results give some insight to find a solution to mitigate extreme low pressures in the draft tube.

Finally, simulations of water column separation in tailrace tunnel resulting from pump-turbine shutdown in generating mode show that the pressure rise resulting from the vapour cavity collapse may reach extreme amplitudes jeopardizing the power plant integrity. The water column separation model implemented in SIMSEN, combined with the already existing hydraulic machines and components models allows estimating the severity of such unwanted event.

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