

# 1D numerical study of a multistage centrifugal pump during transient operation

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**Abstract.** Hydropower plants (HPPs) are acknowledged to be fundamental assets to provide generation flexibility to power systems hosting substantial amount of stochastic renewable energy sources. Within this context, providing this flexibility enforces HPP units to cross-over transient operating conditions or to operate in off-design conditions. The evaluation of the impact that such transients have on the hydraulic machines is a fundamental step to assess the operating costs in terms of lifetime reduction of the machine structural integrity.

When these assessments are performed on experimental test-rigs featuring a machine's reduced scale models, testing transient regimes may potentially trigger harmful pressure profiles along the piping system as well as resonant phenomena on the test-rig. Simulating how the several state variables evolve during the experimental tests allows for the prediction of damaging circumstances and, eventually, prevent performing experimental tests in detrimental conditions.

In this regard, this paper illustrates a numerical method able to reliably reproduce the experimental test-rig behavior while preserving the consistency of the scaled similarities with respect to a specific case study HPP: the Veytaux I pumped-storage hydropower plant located in Switzerland.

Moreover, in this paper, the developed 1D numerical model of the EPFL Technological Platform for Hydraulic Machines experimental test-rig is presented since some of the characteristic transient sequences of the studied HPP are simulated on this model and compared to the prototype scale 1D numerical results.

## 1. Introduction

Since the introduction of electricity industry liberalisation in the '90s, the vertical integration of the electricity supply has been gradually replaced with a reorganisation of the industry around markets. In this market regulated structure, the electricity is traded according to different auctioning mechanism in dedicated market places [1] acting at different time scales. In line with this industry organisation around markets, system

operators generate an unconstrained electricity generation schedule for the trading day by accepting bids provided by the market participants in the pool market. However, system operators must take into account several issues that may arise, as demand forecasts errors, constraints imposed by contracts negotiated in the bilateral market or the stochastic nature of the increasingly deployed non-dispatchable renewable power generation. As a consequence, a constrained schedule has to be built, and redispatching measures become of vital importance to guarantee the stability of the grid. Within this new operational context, transient hydraulic operating conditions are induced by the role that hydropower plants (HPPs) have when providing frequency replacement reserves and intra-day redispatch. These services provided to the power system enforce the hydraulic machines to face frequent start and stop sequences. Start and stop sequences are known to induce dynamic loads and largely excite the machines' structures. In relation to this, the drastic increase contemplated in the near future of the need for these reserves and of their exploitation in transient regimes, such as start up and stop sequences, is frequently illustrated in the literature. Various technical reports clearly show the trend of the penetration of important amount of renewable energy resources (RER) in power systems in different geographical regions [2, 3, 4, 5, 6].

In the case of centrifugal multistage pumps, widely employed in pumped-storage HPPs, dynamic loads linked to these operating conditions can have very disparate origins and causes. In [7], Florjancic and Frei discussed a complete illustration of several sources of forced vibrations detectable on a centrifugal pump. These vibrations can be separated and classified according to their nature: hydraulic (generally predominant in pumps) and mechanical induced excitation forces. Regarding the introduced hydraulic sources, pumps typically have to deal with vibrations caused by vane passing forces: these interactions between rotor and stator, extensively studied also by Berten et al. [8], are strongly dependent on the physical gap between the impeller outlet diameter and the volute or diffuser inlet diameter, as well as on the number of impeller's and diffuser's vanes. Recirculation, separation, rotating stall, cavitation and hydraulic unbalances — due to deviation from the rotational symmetry of the flow through the impeller channel and hard to be distinguished from mechanical unbalances with a frequency analysis — are also phenomena that may induce relevant structural loads on a centrifugal pump.

To the best of the authors' knowledge, experimental works studying dynamic responses of multistage pumps at prototype scale, or for reduced scale models, are very few in the literature (in contrast with the large amount of references on prototype and models of pump-turbines or reaction turbines). In particular, the dynamic response of a pump-turbine impeller has been analyzed at prototype scale by Egusquiza et al. [9], and for a reduced scale model by Escaler et al. [10], where the effect of the surrounding fluid on the runner's modal behavior has been studied. Measurements performed with an instrumented hammer and accelerometers highlight the lower fluid-structure system natural frequencies and the higher damping ratio for runner completely immersed in water, both caused by the added mass of the fluid. A failure analysis of a pump-turbine impeller revealing fatigue damages presented by Egusquiza et al. [11] also provided

useful information regarding its dynamic response, relating it to the high head machine characteristics and to the detected excitation frequency.

The several dynamical loads and eventual damages that can appear during the off-design and transient operating conditions of a multistage pump are the reason why testing a reduced scale model is well suited to evaluate the operational cost of the full-scale machine. In particular, experimental studies on reduced scale models of hydraulic machines instrumented with strain gauges on-board the runner are advanced procedures to evaluate the structural impact on the full-scale hydraulic machine. However, the dynamics of the involved test-rig may present some unexpected features that may induce deleterious effects on its structure when testing specific operating conditions. In fact, the influence of standard and abrupt transient regimes on structural components of HPPs is well known and, despite the significant distinction between HPPs and experimental test-rigs configurations, test-rigs could experience similar conditions. Concerning the HPP case study of this same paper (see Section 2), Nicolet et al. [12] analyzed the influence of a new designed surge tank on the pressure fluctuations induced in several locations of the piping system due to various transient scenarios occurring for different working configurations. Among them, standard and emergency shut downs in pumping mode, also investigated by Lippold and Hellstern [13] for this same HPP and by Jaberg et al. [14] for the Austrian pumped-storage HPP Kops II, revealed substantial head fluctuations in the hydraulic circuit, which may also arise during a reduced scale model experimental campaign, leading to detrimental complications. In relation to this idea, Yang et al. illustrated in [15] the design of an experimental platform reproducing a pumped-storage HPP in transient operating conditions, the aim of which is to conduct a fundamental study of the operating stability of hydropower generating systems by applying a physical model experiment and a theoretical analysis. In [16], Trivedi et al. also performed experimental campaigns, on a test rig featuring a Francis turbine reduced scale model, to investigate the transient pressure fluctuations during start and stop sequences and to classify the structural damages induced by start-up and shutdown sequences on the runner blades.

The model-based prediction of the various state variables during the experimental tests is, therefore, a worthwhile choice to prevent the risk of performing experimental tests in such detrimental conditions.

In view of the above, the aim of this work is to develop a numerical model able to: (i) reliably reproduce experimental test-rigs working conditions and (ii) ensure that the studied HPP characteristics can be represented according to the scaled similarities. For this purpose, a 1D numerical model of one of the test-rigs of the EPFL Technology Platform for Hydraulic Machines has been developed. The paper introduces in Section 2 the case study HPP, followed in Section 3 by the illustration of the EPFL-PTMH test-rig, the implementation of the test-rig's numerical model within the SIMSEN simulation environment [17] and the studied transient operations. In Section 4, the results of the simulated transient regimes and a comparison with the results achieved by the simulations performed on the same transient sequences at prototype scale are presented.

The comparison between the developed model and a prototype scale model (strongly consistent with on-site measurements, see [12] for further details) of the study case HPP enabled to carry out a validation of the test-rig numerical model in terms of scaled similarities.

## 2. Case study

The study presented in the paper and the developed methodology is applied to one powerhouse of the Forces Motrices Hongrin-Léman (FMHL) pumped-storage power plant located in the western region of Switzerland. The power plant features two powerhouses, each one with an installed power of 240 MW. In the studied powerhouse, known as Veytaux I, four ternary units are equipped with the studied multistage pump. The power plant exploits the available head between the Hongrin Reservoir (maximum water level of 1255 m a.s.l.) and the Léman Lake (mean water level of 372 m a.s.l.). Each horizontal axis ternary unit of the powerhouse features an installed power of 60 MW. In pumping mode, a nominal head of 847 mWC for a maximum discharge rate of  $6 \text{ m}^3 \text{ s}^{-1}$  are considered. The 5-stage pump works at a nominal rotating speed of 600 rpm. It is important to remark that the powerhouse also features an auxiliary pump used to fill the gap between the lake level and the HPP pump location.

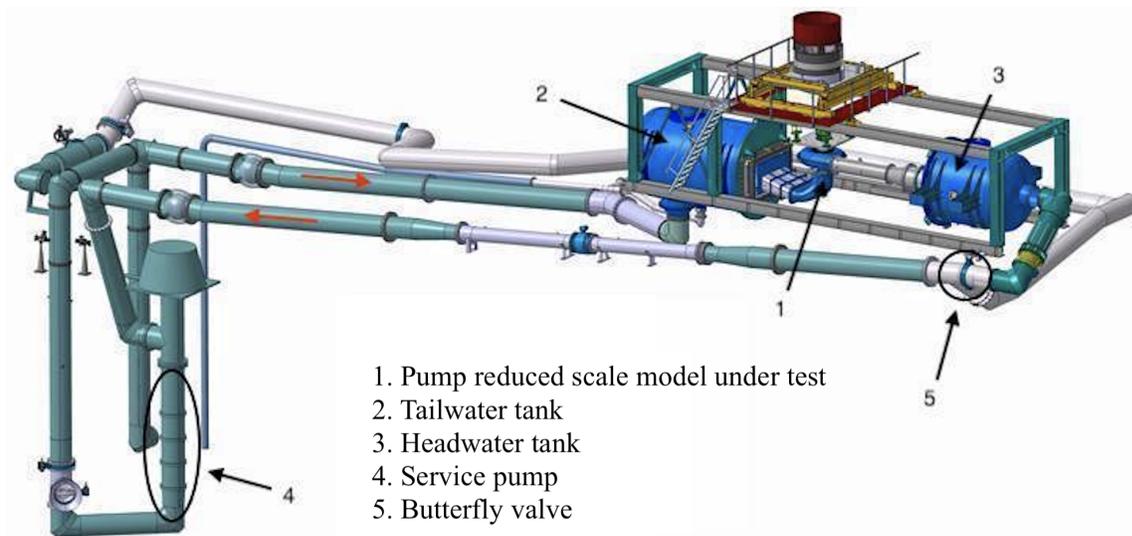
## 3. Methods

### 3.1. EPFL-PTMH test-rig

At the EPFL Technology Platform for Hydraulic Machines (PTMH), experimental tests on a reduced scale model of the multistage pump have been planned. The corresponding multistage pump model is instrumented with strain gauges in order to evaluate the mechanical stresses developed during each tested operating condition. The test-rig is shown in Figure 1.

The reduced scale model of the pump features only three stages instead of five, and it is designed with a diameter such that, given the admissible head on the test-rig, geometrical and Froude similarities can be ensured at each experimental condition. Froude similarity is considered since during transient operating conditions the development of multi-phase flows can strongly affect the transposability of the reduced scale model results. An insight into the transposition from model to prototype scale fulfilling Froude similarity is given by Nicolet et al. in [18]. The maximum admissible rotating speed for the tested model on the test-rig is set at 1500 rpm. In the shown test-rig, the tested model (in Figure 1, cf. item nr.1) is scheduled to be implemented between the two tanks, before the spiral case. Adjusting the pressure in the tailwater tank (cf. item nr.2) allows for a correct scaled similarity with respect to the cavitation number  $\sigma$ . The Froude number and the cavitation number are defined as follows:

$$Fr = \sqrt{\frac{H}{D}} \qquad \sigma = \frac{NPSH}{H} \qquad (1)$$



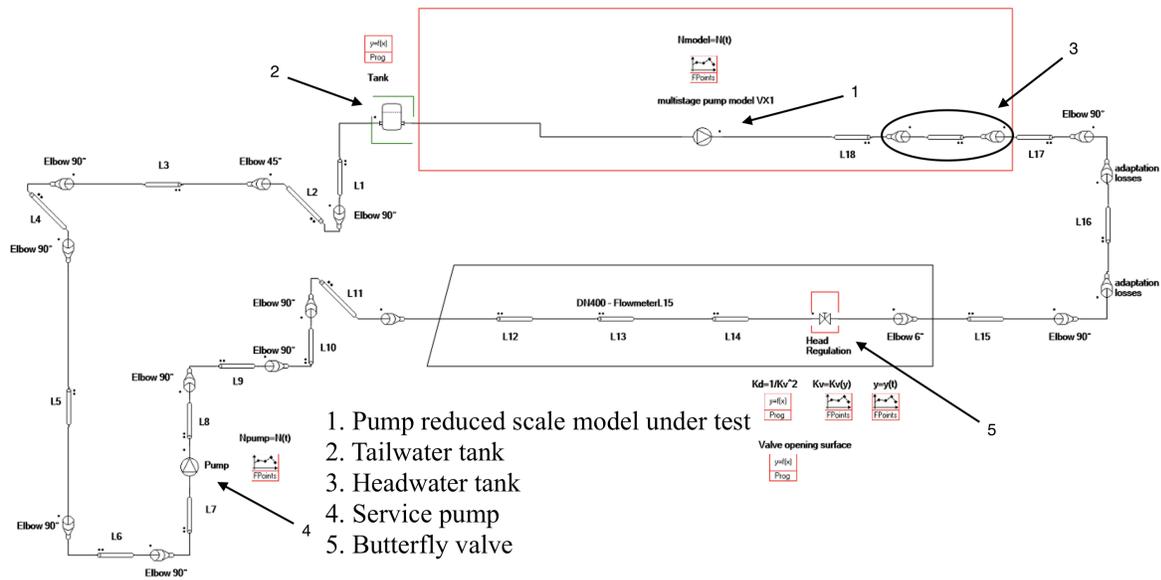
**Figure 1:** EPFL-PTMH test-rig. Red arrows indicates the direction of the water discharge, circulating in the turquoise-coloured piping system.

with  $H$  being the head in m,  $D$  the reference diameter in m and  $NPSH$  defined as the Net Pressure Suction Head, in m. The headwater tank (cf. item nr.3) behaves as a pipe instead. The 900 kW service pump (cf. item nr.4) is used to balance and dissipate the head generated by the reduced scale model. In the high pressure section of the piping system, between the reduced scale model and the service pump, the maximum admissible head is of 160 m. However, the service pump capability of head dissipation is limited to approximately 100 m, which turns out to be the limiting factor regarding the acceptable head in the piping system. In this hydraulic circuit portion, a butterfly valve (cf. item nr.5) is used to control the discharge: this manipulation allows for the implementation of various transient regimes such as standard start up and shut down sequences.

### 3.2. 1D modeling

The test-rig described above has been numerically implemented in the SIMSEN software [17]. Within SIMSEN, a complete model of an hydraulic system can be built, specifying the particular properties of every element in it. The aim of the software is to be able to simulate the dynamic behavior of the system depending on the operating condition introduced by the user. Both stationary and transient regimes can be simulated. In this framework, the implemented model is a closed loop hydraulic circuit. The 1D model is shown in Figure 2. The same five items highlighted in Figure 1 are also indicated.

The types of element needed for the test-rig model design are very few. Every straight circular cross sectioned pipe of the hydraulic circuit and the pressurized headwater tank can be implemented as pipe elements, whereas the elbows and the enlargement or restriction of the piping system's passage section have been modeled



**Figure 2:** 1D model implementation of the EPFL-PTMH test rig in SIMSEN software.

as specific singular losses. The reduced scale model of the HPP multistage pump, the service pump in dissipation mode and the discharge control valve are defined by their physical properties and by the suited operating conditions. They are governed by the logical and mathematical functions elements (see [17] for further details), the purpose of which is to implement the relevant transient sequences of the test-rig in the numerical model.

*3.2.1. Similarity and scaling laws* The definition of the key values of the reduced scale model SIMSEN elements relies on the Froude similarity needed to ensure the hydraulic similarity during the experimental tests. The imposed constraints define the admissible head in the piping system and the rotating speed of the generator, used to regulate the rotational speed of the model.

Given these two limits, the head provided by the model at the nominal operating condition has been set at 90 m. This value results from the trade-off between a high head testing condition, leading to a lower rotational speed of the generator and implying the advantage of having a larger diameter for the reduced scale model, easier to equip with the strain gauges instrumentation, and the physical limit of the test-rig, namely the service pump dissipation capacity, set at 100 m. It is worth to recall the fact that the model features three stages instead of five. This implies that the correct scaling factors have to be defined relating the nominal head during testing conditions to the prototype nominal head of 847 m multiplied for a factor of 3/5.

Behind the geometric, Froude and cavitation similarities, testing a reduced scale model requires both kinematic and dynamic similarity with the prototype scale machine. For hydraulic machines, this translates in a constraint applied to the model and prototype characteristic curves of both the dimensionless IEC Machine Factors for

discharge and torque, as function of the speed factor. These three factors are defined as follows:

$$n_{ED} = \frac{nD}{\sqrt{E}} \quad Q_{ED} = \frac{Q}{D^2\sqrt{E}} \quad T_{ED} = \frac{T}{\rho D^3 E} \quad (2)$$

with  $E = gH$  being the specific energy in  $\text{J kg}^{-1}$ ,  $n$  the rotating speed in  $\text{rad/s}$ ,  $Q$  the discharge flow rate in  $\text{m}^3 \text{s}^{-1}$ ,  $T$  the mechanical torque in  $\text{Nm}$  and  $\rho$  the water density in  $\text{kg m}^{-3}$ .

*3.2.2. Service pump in dissipation mode* The performance of reduced scale model tests of an hydraulic turbine (or pump) satisfying the similarity laws with the full-scale HPP, requires that the characteristic head and discharge of the full-scale HPP are transposed to the test-rig by respecting the previously mentioned non-dimensional parameters. On the PTMH test-rig, the dissipation of the head provided by the multistage pump model can be performed either by needle valves or running the service pump in opposition to the flow direction. Since several transient regimes are expected to be tested, reproducing the realistic power plant's constant head implies the need of a parallel regulation of the tested model and the dissipation device. An accurate time-dependent head regulation can only be performed with the service pump, for which the characteristic curves in terms of speed, discharge and torque factor in a dissipation regime are required.

### *3.3. Transient operation*

Transient regimes in an HPP may induce the generation of severe pressure profiles along the hydraulic system, entailing deleterious consequences. However, these prospective conditions underlie the design constraints of the hydropower plant, since standard and unplanned transient regimes may take place regularly during the plant foreseen lifetime, i.e. in the order of some hundreds per year [6].

Nevertheless, even though typical transient regimes of a specific power plant on a test-rig should give similarly scaled results, some specific features of the test-rig may likely affect the similarity of the testing conditions. As an example, the losses coefficient of the piping system and the valves characteristics may affect the transient evolution of the discharge implying an unpredictable dynamic response dependent on the modal behavior of the test-rig, strongly related to the tested operating conditions as depicted by Landry et al. in [19].

In this work, only few scenarios of the different experimental test campaigns are presented: the standard start and the standard shutdown procedures.

*3.3.1. Standard starting procedure* The represented starting procedure bypasses the acceleration phase of the multistage pump from standstill and shows its behavior once pressurized water already surrounds it and the synchronization procedure has been executed. To fill the gap between the lake level and the HPP pump location and to avoid cavitation inception, an auxiliary pump is used to provide the HPP pump with a

pressurized fluid volume. At the moment of the valve opening, the HPP pump is then surrounded by a water volume that already features a fraction of the total head value required by the plant. Since the nominal head is provided by summing the contribution of both the auxiliary pump and the HPP pump, the amount of head supplied by the HPP pump is lower than the 100% of its nominal head value.

In the numerical simulation, the input parameters are the multistage pump rotational speed, the current difference in elevation of the two reservoirs and the amount of head provided by the auxiliary pump.

*3.3.2. Standard stop procedure* The normal shut down (NSD) procedure performed on a pumped-storage HPP in pumping mode consists in a closing sequence of the spherical valve while maintaining the electrical motor connected to the grid, i.e. maintaining the pump at its nominal rotating speed. Once the valve is completely closed, the motor is disconnected and the pump starts slowing down, until the standstill. The valve closing sequence is composed by two linear slopes: a steeper one along which the valve closes for a first fraction of the full opening, and a more gradual one completing the sequence.

## 4. Results

The results shown in this Section take into account the time scale between the prototype and the reduced scale model defined according to the Froude time scaling, described as follows:

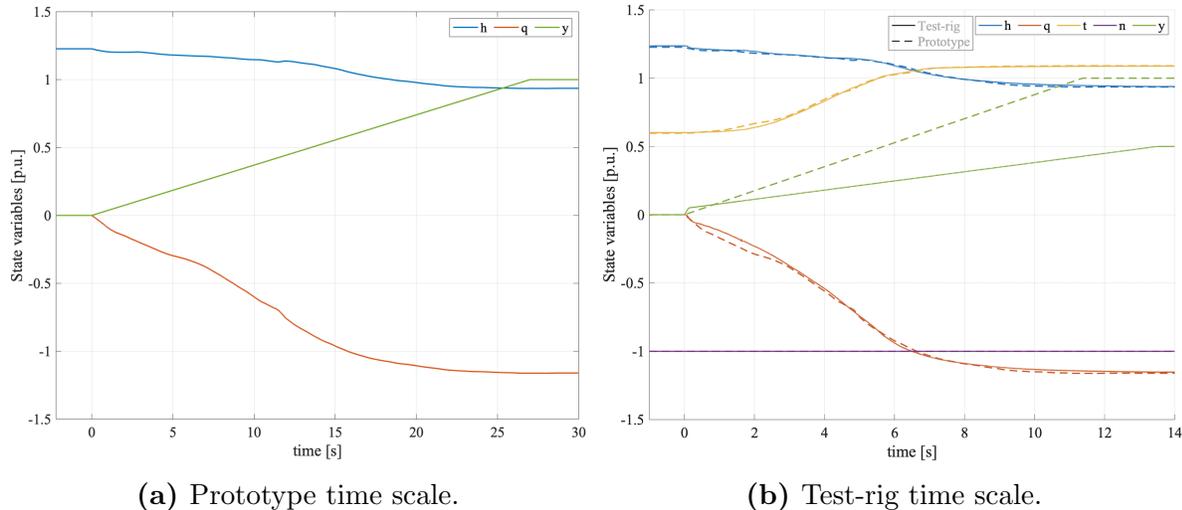
$$\frac{t_P}{t_M} = \lambda^{1/2} \quad (3)$$

with  $t_P$  the prototype time scale,  $t_M$  the reduced scale model time scale and  $\lambda$  the geometrical scale ratio  $\lambda = \frac{D_P}{D_M}$ , with  $D_P$  and  $D_M$  the prototype and the model scale reference diameter respectively. The selected geometrical scale ratio  $\lambda = 5.64$  leads to a time scale ratio of  $\frac{t_P}{t_M} = 2.37$ . To simplify the visualisation in one single graph, the studied state variables are plotted in per-unit.

In Figure 3a, the standard starting procedure of the multistage pump at the prototype scale is shown, where the linear opening (green line) takes place in 27 seconds. As described in Section 3.3.1, once the valve is completely open and the discharge (red line) reaches its steady value, the provided head (blue line) is below the nominal value. The reservoirs at prototype scale have been respectively set at 1182 m a.s.l. and at 373 m a.s.l..

In Figure 3b, the same start-up procedure is shown according to the reduced scale model time scale. The prototype state variables are compared with the test-rig state variables. The only state variable which is constant during the simulation is the impeller rotating speed (purple line). Due to the intrinsic difference between the losses characteristics of the spherical prototype valve and of the butterfly test-rig valve, the opening sequence of the latter has to be adjusted to fit the prototype discharge trend. An initial step followed by a linear opening until a final value of 50% opening

is implemented. After this value, the singular losses induced by the valve are negligible with respect to the discharge curve.



**Figure 3:** Prototype start-up in Figure 3a. Comparison between prototype and test-rig start-up in Figure 3b.

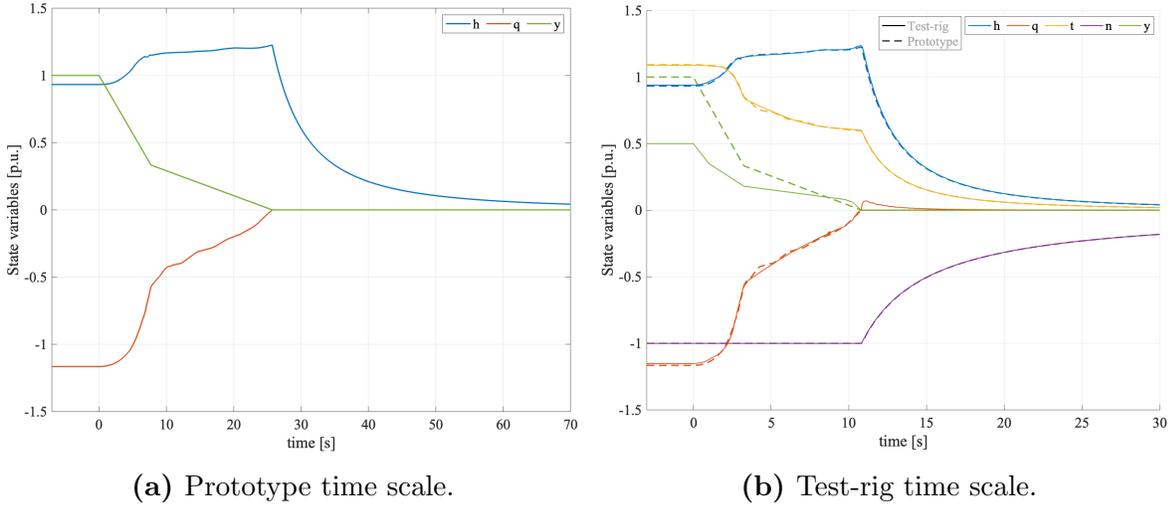
Globally, the trend of the state variables computed on the test-rig numerical model are consistent with the trend of the state variables computed at prototype scale. The adjustment of the service pump in dissipation mode allows for a precise match of the head provided by the multistage pump once the full discharge operating condition is reached.

This strong agreement was expected for steady operating conditions, namely before the valve opening and at full discharge. A maximal deviation between the prototype and test-rig scale computed discharge and mechanical torque of 1% for these two operating conditions is observed, reasonably due to the slight numerical discrepancy given by the transposition of the machine characteristics from the prototype scale to the test-rig scale. However, the observable trends during the transient regimes reveal a remarkable coherence of the developed model with respect to the prototype scale numerical simulations. The larger deviation between the two simulations is detected at the instant just after the change in the test-rig valve opening sequence slope, where both discharge and mechanical torque deviate for a maximum of about 30% and 7% respectively.

The Figure 4a shows the standard NSD procedure of the multistage pump at the prototype scale described in Section 3.3.2. Two thirds of the full closing procedure is performed along the first slope of the sequence split, whereas the remaining closing procedure takes part along the second linear slope. The reservoirs at prototype scale have been respectively set at 1185 m a.s.l. and at 373 m a.s.l.

As for the start-up procedure, the opening sequence of the test-rig valve has to be adjusted to fit the prototype discharge trend. An additional slope split during the two

third closing procedure as well as a final step in the valve sequence are needed.



**Figure 4:** Prototype NSD in Figure 4a. Comparison between prototype and test-rig NSD in Figure 4b.

As it can be observed in Figure 4b, the consistency demonstrated in the case of the start-up procedure is confirmed for the NSD. Following the rotating speed deceleration trajectory by means of the test-rig generator allows for a precise reproduction of the hydraulic head and mechanical torque profiles.

Concerning the steady state initial condition, a maximal deviation lower than 1% is visible for the computed state variables. As stated in the start-up outcomes analysis, it is reasonable to attribute this deviation to the slight numerical discrepancy consequent to the machine characteristics transposition. A larger deviation between the computed trend for the mechanical torque is noticed during the transient regime, just after the change in the slope of the prototype closing valve trajectory, with a deviation peak lower than 4%. The reason is the oscillating behavior of the prototype discharge, following the closing slope change, that can not be reproduced with a realistic adjustment of the test-rig butterfly valve and that results, therefore, in a difference of about 10%.

It is interesting to remark the minor opposite flow computed with the test-rig numerical model just after the pump deceleration onset. This discrepancy between the two models is due to the different valve emplacements in the hydraulic circuit. Unlike the real scale HPP, where the valve is located at the pump outlet, the test-rig valve is located in the middle of the high-pressure circuit. For this reason, when the deceleration procedure starts in the test-rig, an important amount of pressurized water mass is located between the reduced scale model and the valve, inducing the development of a small counter flow, becoming then rapidly almost undetectable after less than 5 seconds.

## 5. Conclusion

This article presents a numerical method to predict the dynamic behavior of the PTMH test-rig for experimental campaigns investigating centrifugal pumps during transient operating conditions. A multistage pump reduced scale model and different scenarios of transient sequences are considered. The test-rig numerical model elements are implemented by respecting the similarity laws required to reliably reproduce the prototype behavior for the tested operating conditions.

The calibration of the simulated hydraulic head on the test-rig numerical model allows for a close reproduction of the test-rig state variables trends when compared to the prototype scale. These parallel simulations validate the accuracy of the test-rig numerical model, which can, therefore, be employed as a trustworthy approach to prior investigate the impact that specific operating conditions may have on the test-rig structural health during a particular experimental campaign.

An increased accuracy of the numerical transposition of the hydraulic machine characteristics as well as the use of a discharge control valve with a characteristic closer to the prototype valve may further decrease the detected discrepancy between the computed state variables trend.

## 6. Acknowledgement

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